LogIQ GC 2025-26

Maths and Physics Club, IIT Bombay

1st November 2025

Read carefully before starting the paper:

- You and your partner have 120 minutes to answer all the questions.
- You are required to give answers as directed by the questions in the space provided in the Answer Booklet. Some questions are objective while some are subjective.
- This is a closed-book, closed-internet test and you're advised not to seek any help in answering the questions. Discuss ONLY with your partner, nobody else.
- Answer the questions only on the answer sheet which you'll be provided with. You will be given blank pages separately for rough work.
- There are **12** questions in section 1, **8** questions in section 2 and **4** questions in section 3. There are a total of **13** pages in this question paper (including this page).
- Each question in section 1 is worth **5 marks**, in section 2 is worth **10 marks**, and in section 3 is worth **15 marks**. The full paper is worth **200 marks**.
- There is no negative marking.
- Non-programmable calculators are allowed.

Hostel Number: ______________________________

Team Name: ______________________________

Name of Candidate 1: ______________________________

Name of Candidate 2: ______________________________

Roll Numbers of both: ______________________________

Section 1: Easy

1. What is the sequence of moves for the rook to kill all pawns? (Every move must kill one pawn)

2. We have 2025 1s written on a board in a line. We randomly choose a strictly increasing sequence from $1, 2, \ldots, 2025$ such that the last term is 2025. If the chosen sequence is $a_1, a_2, \ldots, a_k$ (k is not fixed), then at the i^{th} step, we choose the first a_i numbers on the line and change the 1s to 0s and 0s to 1s. After k steps are over, we calculate the sum of the numbers on the board, say S. What is the expected value of S?

3. Given a graph, in one move you can pick a cycle, add a new vertex and connect all edges from the vertex to all vertices of the cycle, then delete the edges of the cycle. Prove that this is finite.

4. (a) A coin is flipped until you get three consecutive heads. What is the expected number of flips?

   (b) Which sequence of coins - HHTT or HTHT will require lesser expected number of flips to achieve?

5. Albert and Bernard just became friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates:

   1. May 15, May 16, May 19
   2. June 17, June 18
   3. July 14, July 16
   4. August 14, August 15, August 17

   Cheryl then tells Albert and Bernard separately the month and the date of her birthday, respectively.

   Albert: I don't know when Cheryl's birthday is, but I know that Bernard doesn't know too.
   Bernard: At first, I didn't know when Cheryl's birthday is, but I know now.
   Albert: Then I also know when Cheryl's birthday is. So when is Cheryl's birthday?

6. Prove that for every positive integer k there exists a positive integer solution to the equation

$$x_1!x_2!\cdots x_k! = y!$$

   with $x_i \geqslant 2025$ for all i.

7. Shivansh starts with the number 1, and at each step he can multiply his current number with any number more than 1, such that the number remains less than or equal to 20. If he picks randomly between all the options available to him at each step, what is the expected number of steps he is able to do before he can no longer multiply.

8. If $d_1, d_2, , \ldots, d_9$ are distinct decimal digits (from amongst $0, 1, \ldots, 9$), such that

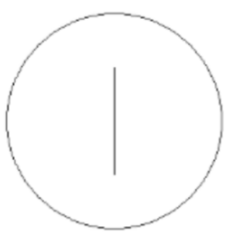
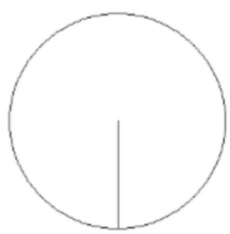
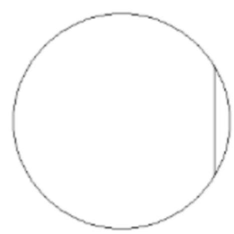
$$d_1||d_2||d_3||d_4 + d_2||d_3||d_4 = d_5||d_6||d_7||d_8||d_9$$

find all the digits $d_1, d_2, \ldots, d_9$.

9. How many ways are there to line up 2025 girls (all of different heights) in a row so that no girl has a shorter girl both in front of and behind her?

10. All edges of an n-complete graph are being deleted with probability $\frac{1}{2}$. Then what is the expected number of isolated vertices finally?

11. In a unit circle, where should you draw a unit line segment, to maximize the probability that it will be intersected by a random chord? The chord is drawn by connecting two uniformly random points on the circle.

A  B  C 

12. A three-digit number has its digits shuffled in every possible way, forming five new distinct numbers (different from the original). The sum of these five new numbers is 2022. What is the original number?

Section 2: Medium

13. Show that $(\sqrt{\text{PASSION}}) = KISS$ has a unique solution in base 10. That is, find the unique correspondence between the letters P, A, S, I, O, N, K and seven of the digits 0, 1,..., 9 such that the equation holds.

14. You have a $2m$ by $2n$ grid of squares coloured in the same way as a standard checkerboard. Find the total number of ways to place mn counters on white squares so that each square contains at most one counter and no two counters are in diagonally adjacent white squares.

15. Each point A in the plane is assigned a real number $f(A)$. It is known that $f(M) = f(A) + f(B) + f(C)$, whenever M is the centroid of a non-degenerate, non-equilateral triangle ΔABC. Prove that $f(A) = 0$ for all points A.

16. Each square of a square grid is colored in one of the three colors: red, yellow or blue, such

that the numbers of squares in each color are the same. If two squares sharing a common edge are in different colors, call that common edge a separating edge. Find the minimal number of separating edges in the grid.

17. There is a table with n rows and 18 columns. Each of its cells contains a 0 or a 1. The table satisfies the following properties:

    1. Every two rows are different.
    2. Each row contains exactly 6 cells that contain 1.
    3. For every three rows, there exist a column so that the intersection of the column with the three rows (the three cells) all contain 0.

    What is the greatest possible value of n?

18. Shivansh has a biased coin with a p $(\neq 0.5)$ chance to return heads and $1 - p$ chance for tails each time it is tossed, where p is a random uniform variable between $[0, 1]$ excluding 0.5. The coin is tossed n times in front of him, and he has to guess what the result will be each time before it is tossed. If he gets 1 dollar for each success and loses 1 dollar for each fail, let $E(W)$ be the expected amount of his earnings. (He doesn't know the value of p.)

    1. Find
    $$\lim_{n\to\infty} \frac{E(W)}{n}$$
    2. For $n = 100$, give a lower bound on $E(W)$. The better the estimate, the higher the marks but any incorrect reasoning has a large penalty.

19. Abhinav is a farmer who never went to school. To sell the (heavy) apples he produced, he has to weigh them using a traditional balance scale (not a spring scale). Each apple weighs an integer number of kilograms, up to a maximum of 240 kg. Occasionally, his old metal weights would get rusty, and he'd need to buy new ones from the hardware store. At the store, weights are sold in whole-number denominations - 1 kg, 2 kg, 3 kg, and so on, with each weight costing the same amount. If he wants to be able to weigh any whole-number weight from 1 kg to 40 kg, what is the minimum number of weights he needs to buy, and which specific weights should he choose?

20. A stick is randomly broken into 3 pieces. What are the expected lengths of the shortest, middle and longest pieces?

Section 3: Hard

21. Alice plays a game where she aims to reach a score of n to win. If her score reaches 0, she loses. The probability that she wins a single round depends on her current score x, given by a function $p(x)$. She either wins or loses each round; there are no draws.

    Suppose that Alice starts with a score of r, and that winning a round gains Alice $+1$ point, losing causes her to lose 1 point, find the probability that Alice eventually wins the game. (*Try solving for* $p(x) = \frac{x+1}{2x+1}$)

22. Each positive integer on the number line has a traffic light that can be green, yellow, or red. A bug starts at position 1 and moves according to these rules:

    - If the current light is green, the bug changes it to yellow and moves one step right.
    - If the current light is yellow, the bug changes it to red and moves one step right.
    - If the current light is red, the bug changes it to green and moves one step left.

    The bug continues moving until it either falls off the left end of the number line (moves left from position 1) or goes forever to the right.

    Once the first bug finishes its journey, a second bug is released at position 1, starting from the final light configuration left by the first bug. Then a third bug is released after the second finishes, again using the state of the lights left behind.

    Prove that if the second bug eventually falls off the left end, then the third bug will necessarily travel indefinitely to the right.

23. The names of 100 prisoners are placed in 100 wooden boxes, one name to a box, and the boxes are lined up on a table in a room. One by one, the prisoners are led into the room; each may look in at most 50 boxes, but must then leave the room exactly as he found it and is permitted no further communication with the others. The prisoners have a chance to plot their strategy in advance, and they are going to need it, because unless every single prisoner finds his own name all will subsequently be executed.

    (a) Find a strategy that gives the prisoners a decent chance of survival.

(b) Now one person is allowed to look at all the boxes beforehand but cannot say anything to the other prisoers about the contents of the boxes. This person is allowed to swap at most 2 boxes. What is the strategy now and what is the chance of survival now?

24. Let $S = \{1, 2, 3, \ldots, 100\}$ be a set of 100 distinct numbers written in some unknown order $\pi : S \to \{1, 2, \ldots, 100\}$, where π is a permutation of S. At each query, one may select any subset $Q \subseteq S$ of size $|Q| = 50$ and obtain the total order (i.e., the relative ordering) of the elements in Q according to π. What is the minimum number of such queries required to determine the complete permutation π?